Supersymmetric black holes in 2D dilaton supergravity: baldness and extremality

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Abstract
We present a systematic discussion of supersymmetric solutions of 2D dilaton
supergravity. In particular those solutions which retain at least half of the
supersymmetries are ground states with respect to the bosonic Casimir
function (essentially the ADM mass). Nevertheless, by tuning the prepotential
appropriately, black-hole solutions may emerge with an arbitrary number
of Killing horizons. The absence of dilatino and gravitino hair is proved.
Moreover, the impossibility of supersymmetric dS ground states and of
nonextremal black holes is confirmed, even in the presence of a dilaton. In
these derivations, knowledge of the general analytic solution of 2D dilaton
supergravity plays an important role. The latter result is addressed in the more
general context of gPSMs which have no supergravity interpretation.

Finally it is demonstrated that the inclusion of non-minimally coupled
matter, a step which is already nontrivial by itself, does not change these
features in an essential way.

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1. Introduction

In the mid-1990s, during and after the 'second string revolution', BPS (Bogomolnyi–Prasad–Sommerfield [1]) black holes (BHs) [2, 3] have attracted much interest because in particular they allow us to derive the BH entropy by counting D-brane microstates exploiting string dualities (for reviews cf e.g. [4]). We define a BPS BH as a supergravity (SUGRA) solution respecting half of the supersymmetries and exhibiting at least one Killing horizon in the bosonic line element.

A key reference for the properties of supersymmetric solutions in 2D dilaton SUGRA is the work of Park and Strominger [5]. As shown by these authors for the SUGRA version

of the CGHS model [6], as well as for a SUGRA extended generalized 2D dilaton theory, a certain vacuum solution can be defined with vanishing fermions. A specific solution, which still retained one supersymmetry, was constructed for the CGHS-related model. In the generic case the existence of such a solution was proved, but it was not constructed.

Until quite recently a systematic study of *all* supersymmetric solutions in 2D dilaton SUGRA theories has not been possible. Recently two of the present authors have shown [7, 8] that the superfield formulation of [5] can be identified with the one of a certain subclass of graded Poisson–Sigma models (gPSMs). General gPSMs are fermionic extensions of bosonic PSMs which, in the present case, are taken to be 2D dilaton theories of gravity [9]. This subclass of gPSMs has been dubbed minimal field SUGRA (MFS) in [8]. The important consequence of this equivalence is that the known analytic solution in the MFS formulation [8] represents the full solution for dilaton SUGRA [5], including all solutions with nonvanishing fermionic field components.

This permits us to attack in a systematic manner the problem of 2D SUGRA solutions which retain at least one supersymmetry, the main goal of our present work. It turns out that, although some information can be obtained from the symmetry relations, knowledge of the full solution is a necessary input. This is of particular importance regarding the eventual existence of fermionic hair for BHs[1].

In doing this we are also able to incorporate SUGRA invariant matter, where a special previous model [11] is extended to generalized dilaton theories and also transcribed into the more convenient MFS formulation. Although no general analytic solution is possible when matter is included, we show that there are no nonextremal BH solutions of 2D SUGRA respecting half of the supersymmetries. This result is not unexpected: 'pure' SUGRA—i.e. not deformed by a (super-) dilaton field—does not permit nonextremal supersymmetric BH solutions according to a simple but elegant argument[2] due to Gibbons [2]. Actually, in 2D *dilaton* SUGRA a more direct proof is possible, which avoids the continuation to Euclidean signature: first, we show that the body of the Casimir function (a quantity in gPSM theories related to the energy of the system) has to vanish and then we prove that these ground-state solutions cannot provide simple zeros of the Killing norm. *En passant* all solutions respecting at least half of the supersymmetries are classified including those of [5].

The paper is organized as follows: after a short review of MFS models (section 2), all ground states with unbroken supersymmetry are discussed in section 3, which turn out to be constant dilaton vacua. Such vacua appear nontrivially in some, but not all dilaton theories. Section 4 is devoted to a classification of solutions respecting half of the supersymmetries. All of them imply a vanishing body for the bosonic Casimir function. In section 5, it is demonstrated that for such ground states the Killing norm has to be positively semi-definite and thus only extremal horizons may exist. This statement even can be extended to general gPSM theories. Section 6 discusses the coupling of MFS to matter degrees of freedom and section 7 generalizes the result of section 5 to the one including matter.

The notation is explained in appendix A. Equations of motion and some aspects of the exact solution are contained in appendix B. The subject of appendix C is some key formulae [8] relating MFS to the superfield formulation of 2D dilaton SUGRA [5], which are needed for the construction of the matter couplings.

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[1] An early attempt to prove the validity of this no-hair conjecture for the CGHS model can be found in [10].

[2] A crucial ingredient is the continuation to the Euclidean domain and the observation that the absence of conical singularities enforces boundary conditions of thermal quantum field theory, which are not compatible with supersymmetry [12] implying $T = 0$ and hence extremality.

2. Minimal field SUGRA (MFS)

General 2D dilaton SUGRA can be formulated in terms of a gPSM [9, 13]. Its action with target space variables X^I, gauge fields A_I and Poisson tensor P^{IJ}

$$\mathcal{S}_{\text{gPSM}} = \int_M \mathrm{d}X^I \wedge A_I + \frac{1}{2} P^{IJ} A_J \wedge A_I \tag{2.1}$$

is invariant under the symmetry transformations

$$\delta X^I = P^{IJ} \varepsilon_J \qquad \delta A_I = -\mathrm{d}\varepsilon_I - (\partial_I P^{JK}) \varepsilon_K A_J \tag{2.2}$$

as a consequence of the graded nonlinear Jacobi identity

$$P^{IL} \partial_L P^{JK} + g\text{-perm}\,(IJK) = 0. \tag{2.3}$$

Not every gPSM may be used to describe 2D SUGRA. A subclass of appropriate models (called minimal field SUGRA, MFS) has been identified in [7, 8] for $N = (1, 1)$ SUGRA. It contains a dilatino χ_α and a gravitino ψ_α, both of which are Majorana spinors ($X^I = (\phi, X^a, \chi^\alpha)$, $A_I = (\omega, e_a, \psi_\alpha)$, $Y = X^{++} X^{--}$):

$$P^{a\phi} = X^b \epsilon_b{}^a \qquad P^{\alpha\phi} = -\frac{1}{2} \chi^\beta \gamma_{*\beta}{}^\alpha \tag{2.4}$$

$$P^{ab} = \left(V + YZ - \frac{1}{2} \chi^2 \left(\frac{VZ + V'}{2u} + \frac{2V^2}{u^3} \right) \right) \epsilon^{ab} \tag{2.5}$$

$$P^{\alpha b} = \frac{Z}{4} X^a (\chi \gamma_a \gamma^b \gamma_*)^\alpha + \frac{iV}{u} (\chi \gamma^b)^\alpha \tag{2.6}$$

$$P^{\alpha\beta} = -2i X^c \gamma_c^{\alpha\beta} + \left(u + \frac{Z}{8} \chi^2 \right) \gamma_*^{\alpha\beta} \tag{2.7}$$

V, Z and the prepotential u are functions of the dilaton field ϕ and obey the relation

$$V = -\tfrac{1}{8}[(u^2)' + u^2 Z]. \tag{2.8}$$

This Poisson tensor leads to the action

$$\begin{aligned}
\mathcal{S}_{\text{MFS}} = \int_{\mathcal{M}} \Bigg(& \phi\,\mathrm{d}\omega + X^a De_a + \chi^\alpha D\psi_\alpha + \epsilon \left(V + YZ - \frac{1}{2} \chi^2 \left(\frac{VZ + V'}{2u} + \frac{2V^2}{u^3} \right) \right) \\
& + \frac{Z}{4} X^a (\chi \gamma_a \gamma^b e_b \gamma_* \psi) + \frac{iV}{u} (\chi \gamma^a e_a \psi) \\
& + i X^a (\psi \gamma_a \psi) - \frac{1}{2} \left(u + \frac{Z}{8} \chi^2 \right) (\psi \gamma_* \psi) \Bigg).
\end{aligned} \tag{2.9}$$

An important class of simplified models is described by the special choice, $\bar{Z} = 0$. Following the nomenclature of [8] it is called MFS_0 and barred variables are used. As can be verified easily the action (2.9) and $\mathcal{S}_{\text{MFS}_0}$ are related by a conformal transformation of the fields ($Q'(\phi) = Z(\phi)$)

$$\phi = \bar{\phi} \qquad X^a = \mathrm{e}^{-\frac{1}{2} Q(\phi)} \bar{X}^a \qquad \chi^\alpha = \mathrm{e}^{-\frac{1}{4} Q(\phi)} \bar{\chi}^\alpha \tag{2.10}$$

$$\omega = \bar{\omega} + \frac{Z}{2} \left(\bar{X}^b \bar{e}_b + \frac{1}{2} \bar{\chi}^\beta \bar{\psi}_\beta \right) \qquad e_a = \mathrm{e}^{\frac{1}{2} Q(\phi)} \bar{e}_a \qquad \psi_\alpha = \mathrm{e}^{\frac{1}{4} Q(\phi)} \bar{\psi}_\alpha. \tag{2.11}$$

The symmetry transformations (2.2) for fermionic ϵ with (2.4)–(2.7) become

$$\delta\phi = \frac{1}{2}(\chi\gamma_*\varepsilon) \tag{2.12}$$

$$\delta X^a = -\frac{Z}{4}X^b(\chi\gamma_b\gamma^a\gamma_*\varepsilon) - \frac{\mathrm{i}V}{u}(\chi\gamma^a\varepsilon) \tag{2.13}$$

$$\delta\chi^\alpha = 2\mathrm{i}X^c(\varepsilon\gamma_c)^\alpha - \left(u + \frac{Z}{8}\chi^2\right)(\varepsilon\gamma_*)^\alpha \tag{2.14}$$

$$\delta\omega = \frac{Z'}{4}X^b(\chi\gamma_b\gamma^a\gamma_*\varepsilon)e_a + \mathrm{i}\left(\frac{V}{u}\right)'(\chi\gamma^a\varepsilon)e_a + \left(u' + \frac{Z'}{8}\chi^2\right)(\varepsilon\gamma_*\psi) \tag{2.15}$$

$$\delta e_a = \frac{Z}{4}(\chi\gamma_a\gamma^b\gamma_*\varepsilon)e_b - 2\mathrm{i}(\varepsilon\gamma_a\psi) \tag{2.16}$$

$$\delta\psi_\alpha = -(D\varepsilon)_\alpha + \frac{Z}{4}X^a(\gamma_a\gamma^b\gamma_*\varepsilon)_\alpha e_b + \frac{\mathrm{i}V}{u}(\gamma^b\varepsilon)_\alpha e_b + \frac{Z}{4}\chi_\alpha(\varepsilon\gamma_*\psi). \tag{2.17}$$

By eliminating X^a and the torsion-dependent part of the spin connection a new action (MFDS) in terms of dilaton, dilatino, zweibein and gravitino is obtained[3]

$$\begin{aligned}
\mathcal{S}_{\mathrm{MFDS}} = \int \mathrm{d}^2xe\,\Bigg(&\frac{1}{2}\tilde{R}\phi + (\chi\tilde\sigma) + V - \frac{1}{4u}\chi^2\left(VZ + V' + 4\frac{V^2}{u^2}\right) \\
&-\frac{1}{2}Z\left(\partial^m\phi\partial_m\phi + \frac{1}{2}(\chi\gamma_*\psi^m)\partial_m\phi + \frac{1}{2}\epsilon^{mn}\partial_n\phi(\chi\psi_m)\right) \\
&-\frac{\mathrm{i}V}{u}\epsilon^{mn}(\chi\gamma_n\psi_m) + \frac{u}{2}\epsilon^{mn}(\psi_n\gamma_*\psi_m)\Bigg).
\end{aligned} \tag{2.18}$$

Its supersymmetry transformations read

$$\delta\phi = \frac{1}{2}(\chi\gamma_*\varepsilon) \tag{2.19}$$

$$\delta\chi^\alpha = -2\mathrm{i}\epsilon^{mn}\left(\partial_n\phi + \frac{1}{2}(\chi\gamma_*\psi_n)\right)(\varepsilon\gamma_m)^\alpha - \left(u + \frac{Z}{8}\chi^2\right)(\varepsilon\gamma_*)^\alpha \tag{2.20}$$

$$\delta e_m{}^a = \frac{Z}{4}(\chi\gamma^a\gamma^b\gamma_*\varepsilon)e_{mb} - 2\mathrm{i}(\varepsilon\gamma^a\psi_m) \tag{2.21}$$

$$\delta\psi_{m\alpha} = -(\tilde{D}\varepsilon)_\alpha + \frac{\mathrm{i}V}{u}(\gamma_m\varepsilon)_\alpha + \frac{Z}{4}\left(\partial^n\phi(\gamma_m\gamma_n\varepsilon)_\alpha + \frac{1}{2}(\psi_m\gamma^n\chi)(\gamma_n\gamma_*\varepsilon)_\alpha\right). \tag{2.22}$$

The action (2.18) has been shown to be equivalent [8] to the general dilaton superfield SUGRA of Park and Strominger [5] (cf appendix C).

3. Both supersymmetries unbroken

From the MFS supersymmetry transformations in section 2 one can read off different conditions for solutions respecting full supersymmetry. From (2.12) and (2.16) follows[4] $\chi^+ = \chi^- = \psi_+ = \psi_- = 0$. The two terms in (2.14) are linearly independent and thus $X^{++} = X^{--} = u = 0$ as well. In addition (2.17) implies that the transformation parameters

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[3] Quantities with a tilde refer to the dependent spin connection $\tilde\omega_a = \epsilon^{mn}\partial_n e_{ma} - \mathrm{i}\epsilon^{mn}(\psi_n\gamma_a\psi_m)$, i.e. $\tilde\sigma_\alpha = *(\tilde{D}\psi)_\alpha$ and $\tilde{R} = 2*\mathrm{d}\tilde\omega$. The quantity $\tilde\sigma - \alpha$ is the fermionic partner of the curvature scalar $\tilde{R}$.

[4] Conventions and light-cone coordinates are summarized in appendix A.

must be covariantly constant: $(D\varepsilon)_\alpha = 0$. For a solution where both supersymmetries are unbroken the Poisson tensor (2.4)–(2.7) vanishes identically.

The equations of motion imply that the dilaton ϕ has to be a constant. It is restricted to a solution of the equation $u = 0$. Such constant dilaton vacua (CDV) are, for instance, encountered [14] in the 'kink' solution of the dimensionally reduced gravitational Chern-Simons term [15].

We recall in appendix B that a key ingredient of the solution is the conserved Casimir function. At this point its additive ambiguity can be fixed: supersymmetry covariance requires that solutions respecting both supersymmetries have vanishing Casimir function. This means that eventual additive constants in (B.10), (B.11) are absent.

Positivity of energy would imply $u^2 \geqslant 8Y$ because the Casimir function C_B is related to the negative ADM mass (cf section 5 of [16]). If this inequality is saturated the ground state is obtained. Equation (B.10) in particular implies that all CDV solutions with $u = 0$ have vanishing body of the Casimir function[5].

As an illustration we consider a two-parameter family of models (the so-called 'ab-family') encompassing most of the relevant ones [17]. Among other solutions, BHs immersed in Minkowski, Rindler or (A)dS space can be described. This family is defined by (2.9) with

$$Z(\phi) = -\frac{a}{\phi} \qquad u(\phi) = c\phi^\alpha \qquad \alpha, a, c \in \mathbb{R}. \tag{3.1}$$

Supersymmetry restricts the constant $B = c^2(b+1)/4$ in the potential (2.8)

$$V(\phi) = -\frac{B}{2}\phi^{a+b} \qquad \alpha = \frac{a+b+1}{2} \tag{3.2}$$

to $B > 0$ if $b > -1$ and to $B < 0$ if $b < -1$. The curvature scalar of the ground state geometry is given by

$$R = \frac{bc^2}{2}(\alpha - a)X^{2(\alpha-1)}. \tag{3.3}$$

The Minkowski ground state (MGS) condition[6] reads $\alpha = a$, Rindler space follows for $b = 0$ and (A)dS means $\alpha = 1$. In the latter case supersymmetry restricts the curvature scalar $R = c^2(1-a)^2/2$ to positive values, which in our notation implies AdS.

For fully supersymmetric solutions of the ab-family the only possible values for the dilaton are $\phi = 0$ or $|\phi| = \infty$ (depending on the value of α), unless $c = 0$, in which case the prepotential u vanishes identically.

4. One supersymmetry unbroken

4.1. Casimir function $C_B = 0$

The symmetry transformation $\delta\phi = 0$ of the dilaton from (2.12) implies

$$\chi^+\varepsilon_+ = \chi^-\varepsilon_-. \tag{4.1}$$

The vanishing of (2.14) and (2.16) leads to

$$u\varepsilon_- = -2\sqrt{2}X^{++}\varepsilon_+ \qquad u\varepsilon_+ = -2\sqrt{2}X^{--}\varepsilon_- \tag{4.2}$$

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[5] CDV solutions with $u \neq 0$ must obey $u'/u = -\frac{1}{2}Z$, which leads to $C_S = 0$ while $C_B \neq 0$. Clearly, they cannot respect both supersymmetries.

[6] It means simply that for vanishing bosonic Casimir function the bosonic line element is diffeomorphic to the one of Minkowski space.

$$\varepsilon \gamma_a \psi = -\mathrm{i}\frac{Z}{8}\epsilon_a{}^b e_b(\chi\varepsilon).$$ (4.3)

In (4.2) terms proportional to $\chi^2\varepsilon$ have to vanish as a consequence of (4.1). Equations (4.2) require

$$Y = \tfrac{1}{8}u^2$$ (4.4)

which in turn implies that the body of the Casimir function (B.10) vanishes. In this sense, BPS-like states are always ground states. Note that equation (4.4) remains valid in the case $u = 0$, implying that at least one component of X^a vanishes as well and vice versa.

It is worthwhile emphasizing that (4.4) corresponds to a vanishing determinant

$$\Delta = \det\begin{pmatrix} -2\sqrt{2}X^{++} & -u \\ -u & -2\sqrt{2}X^{--} \end{pmatrix}$$ (4.5)

of the bosonic part of (2.7) (cf [9]). $\Delta = 0$ must hold for any solution that respects at least one supersymmetry.

4.2. Classification of solutions

4.2.1. Vanishing fermions. We first consider the case of vanishing dilatino χ^α and gravitino ψ_α. Then all three quantities u, X^a have to be nonvanishing or otherwise the solution with full supersymmetry of section 3 is recovered. With the conditions (4.2) and hence also (4.4) the variations (2.12)–(2.16) vanish, while (2.17) equal zero represents two differential equations for ε_+, resp. ε_-. By inserting the explicit solution (section 6 of [8]) it can be checked that both are identical. A straightforward calculation, without using any further restrictions on the different variables involved, yields

$$\mathrm{d}\varepsilon_+ + \left(\frac{\mathrm{d}X^{++}}{2X^{++}} - \left(\frac{u'}{u} + \frac{1}{2}Z\right)\mathrm{d}\phi\right)\varepsilon_+ = 0$$ (4.6)

possessing the general solution (Q is defined in (B.12))

$$\varepsilon_+ = \mathrm{e}^{\frac{1}{2}Q}\frac{u}{\sqrt{X^{++}}}\tilde{\varepsilon}.$$ (4.7)

Here $\tilde{\varepsilon}$ is a spinorial integration constant. ε_- is obtained via (4.2). Thus, all solutions without fermion fields, exhibiting one supersymmetry, leave a linear combination of the two supersymmetries unbroken, in agreement with the discussion in [5].

There exists one special case of (4.6) where an even simpler solution exists: if $Z = 0$ (MFS$_0$ in the parlance of [8]) and if in addition u is a constant, the differential equation (4.6) simply says that the symmetry parameter is covariantly constant. This model is generalized teleparallel dilaton gravity. As u is simply a cosmological constant, it drops out in the constraint algebra, and therefore this case has been referred to as rigid supersymmetry in [9].

4.2.2. Nonvanishing fermions and no-hair theorem.
Further supersymmetric solutions are possible for nonvanishing fermions, a situation not considered in [5], but relevant for the question of fermionic hair[7].

Assuming that at least one component of χ_α is different from zero, e.g. $\chi^+ \neq 0$, equation (4.1) can have the chiral solution $\varepsilon_+ = \chi^- = 0$, while $\varepsilon_- \neq 0$ provides the remaining supersymmetry, or it can relate ε_+ to ε_- by means of $\chi^+ \neq 0$ and $\chi^- \neq 0$.

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[7] No-hair theorems are a recurrent theme in BH physics ([18] and references therein).

Solutions of the first type are almost trivial, because they require $u = X^{--} = 0$. Since the dilaton must not be constant (or else the CDV case would be recovered) this implies that u must vanish identically, and not just at a certain value of ϕ. All fermions must be of one chirality, in particular $\varepsilon_+ = \chi^- = \psi_- = 0$. The quantities $X^{++}, e_{++}, \omega, \chi^+$ must be covariantly constant and may contain soul contributions. The dilaton has to fulfil the linear dilaton vacuum condition $\mathrm{d}\phi = \mathrm{const}$. Only e_{--} and ε_- are slightly nontrivial and can be deduced from

$$(De)_{--} = Z\mu e_{--} \qquad (D\varepsilon)_- = \frac{Z}{2}\mu\varepsilon_- \qquad \mu = \left(X^{++}e_{++} + \frac{1}{2}\chi^+\psi_+\right). \tag{4.8}$$

The bosonic line element is flat and obviously the Casimir function is identically zero.

For the remaining class of solutions with *both* components of χ and of ε different from zero the body of the Casimir function still vanishes, but the soul can be nonvanishing as from (2.13) and (B.11):

$$Z = -\frac{u'}{u} \qquad C_S = \frac{1}{32}\,\mathrm{e}^Q u'\chi^2. \tag{4.9}$$

This is equivalent to the MGS condition mentioned in section 3 below (3.3). Thus, only MGS models are allowed and since the solution has to be the ground state, the bosonic part of the geometry is trivially Minkowski space.

Therefore, *a solution with nonvanishing fermions must have a trivial bosonic background* (this feature is true also for the first type). Consequently there exist no BPS BHs with fermionic hair.

There remains a technical subtlety about the nonchiral solutions. As both components of ε are non-zero, (4.2) implies that all interesting cases have u, X^{++} and X^{--} different from zero. Actually, the solutions presented in [8] do not cover this case, because the one for $C \neq 0$ ((6.9)–(6.13) of [8]), depending on C^{-1}, cannot be used in the present case, as the inverse of a pure soul is ill-defined. The solution for $C = 0$ ((6.17)–(6.21) of [8]) depends on an arbitrary fermionic gauge potential $\tilde{A} = -\mathrm{d}f$, which is the gauge potential associated with the additional fermionic Casimir function (equation (6.15) of [8]) appearing in that case. Analysing the equations of motion (e.o.m.-s) (appendix B and cf also [9]) for $C_B = 0$ but $C_S \neq 0$, this fermionic Casimir function no longer appears, but the solution remains valid if the gauge potential $\tilde{A} = -\mathrm{d}f$ obeys the constraint $\chi^2\tilde{A} = 0$.

Finally (2.17) again leads to the differential equation

$$D\varepsilon_+ - \left(\frac{Z}{16}\frac{u^2}{X^{++}}e_{--} - \frac{Z}{2}X^{++}e_{++}\right)\varepsilon_+ = 0. \tag{4.10}$$

Inserting the solution described above all trilinear and higher spinorial terms are found to vanish. Thus, the solution of (4.10) reduces to (4.7).

5. No nonextremal BPS black holes

5.1. MFS models

It has been shown in section 4 that the body of the bosonic Casimir function has to vanish for all solutions respecting at least half of the supersymmetries. We will focus first on BPS solutions where either $X^{++} \neq 0$ or $X^{--} \neq 0$ (or both).

In the bosonic case ground-state solutions $C = 0$ imply for the line element (cf e.g. equations (3.26), (3.27) of [16], $\otimes$ denotes the symmetrized tensor product)

$$(\mathrm{d}s)^2 = 2\mathrm{d}F \otimes (\mathrm{d}r - \mathrm{e}^{Q(\phi)}w(\phi)\,\mathrm{d}F) \qquad \mathrm{d}r = \mathrm{e}^{Q(\phi)}\,\mathrm{d}\phi. \tag{5.1}$$

Obviously, there always exists a Killing vector $\xi^\mu \partial_\mu = \partial_F$ the norm of which is given by $K = -2\mathrm{e}^{Q(\phi)}w(\phi)$. By choosing the function $w(\phi)$ appropriately, nonextremal Killing horizons are possible, e.g. for $Q = 0$, $-2w = 1 - 2m/\phi$ a Schwarzschild-like BH emerges.

In SUGRA, however, (B.12) implies a negative (semi-)definite w and hence the Killing norm K can only have zeros of even degree:

$$K(\phi) = -2\mathrm{e}^{Q(\phi)}w(\phi) = \left(\tfrac{1}{2}u(\phi)\,\mathrm{e}^{Q(\phi)}\right)^2 \geqslant 0. \tag{5.2}$$

Thus, if a Killing horizon exists it has to be an extremal one, which confirms the general proof using thermal field theory arguments [2]. For instance, with $u = 2(\sqrt{\phi} - M)$, $Z(\phi) = -1/(2\phi)$ the line element reads $(\mathrm{d}s)^2 = 2\mathrm{d}F \otimes (\mathrm{d}r + (1 - M/r)^2 \mathrm{d}F)$, which is the two-dimensional part of an extremal Reissner–Nordström BH[8].

One can trivially generalize this result to all ground-state solutions which are not CDV, i.e. non-supersymmetric solutions of 2D dilaton SUGRA with vanishing body of the Casimir function and non-constant dilaton, because the key inequality (5.2) still holds.

Finally, the simpler CDV case $X^{++} = 0 = X^{--}$ shall be addressed. As shown in the previous two sections CDV implies that both supersymmetries are either broken or unbroken. The equations of motion imply a vanishing body of torsion and a constant body of curvature. Thus—as in the bosonic case (cf section 2.1 in [14])—only (A)dS, Rindler[9] or Minkowski spaces are possible. However, supersymmetry provides again an obstruction: curvature is proportional to $(u')^2$ and thus again the dS case, together with the possibility of nonextremal Killing horizons, is ruled out by supersymmetry.[10]

5.2. Generic gPSM gravity

The question of nonextremal BPS BHs may be addressed in a more general context, namely for gPSM gravity that does not belong to the MFS class. Here we consider generic Poisson tensors with local Lorentz invariance implemented as in equation (2.4). In addition the fermionic extension $P^{\alpha\beta}$ must have full rank almost everywhere in the space of solutions with the notable exception of those which still obey equation (4.1) and where consequently the determinant Δ (analogous to (4.5)) vanishes. These solutions will be called BPS because they still respect half of the fermionic symmetries.

In a generic gPSM (2.7) is replaced by $P^{\alpha\beta} = v^{\alpha\beta} + \chi^-\chi^+ v_2^{\alpha\beta}$ with

$$v^{\alpha\beta} = \begin{pmatrix} \sqrt{2}X^{++}(\tilde{u} - \hat{u}) & -u \\ -u & \sqrt{2}X^{--}(\tilde{u} + \hat{u}) \end{pmatrix} \tag{5.3}$$

where u, $\tilde{u}$ and $\hat{u}$ are functions of ϕ and Y [9]. $v_2^{\alpha\beta}$ is determined by the Jacobi identity (2.3). Also in the bosonic potential

$$P^{ab} = \epsilon^{ab}(v(\phi, Y) + \chi^-\chi^+ v_2(\phi, Y)) \tag{5.4}$$

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[8] In dilaton (super)gravity the number and types of horizons can be adjusted by selecting a certain behaviour of the functions w and (to a lesser extent) Q, which enter the Killing norm (5.2). In many cases of physical relevance extremality is induced by tuning of certain charges/constants of motion, but we emphasize that the explicit presence of additional (gauge) fields by no means is necessary for extremality. For instance, the Reissner–Nordström BH can be constructed either from spherically reduced Einstein–Maxwell theory by tuning the two Casimir functions (mass and charge) accordingly, but it is also possible to provide an effective description where one of these constants, the charge, enters as a parameter of the action rather than a constant of motion.

[9] If curvature is nonvanishing the Rindler term can always be absorbed by a linear redefinition of the coordinates $r \to r' = \alpha r + \beta$, $F \to F' = F/\alpha$, $\alpha \neq 0$. For vanishing curvature the Minkowski term can always be absorbed by a similar redefinition.

[10] In fact, there is one very trivial possibility that remains for a CDV solution with vanishing curvature which allows for the existence of exactly one nonextremal Killing horizon: the line element $(\mathrm{d}s)^2 = 2\,\mathrm{d}F \otimes (\mathrm{d}r + br\,\mathrm{d}F)$, $b \neq 0$ contains a nonextremal (Rindler) horizon at $r = 0$. However, this is neither a BPS state nor a true BH solution.

the body is an independent function, while v_2 again follows from the Jacobi identity. Vanishing determinant of (5.3) implies

$$Y = \frac{u^2}{2(\tilde{u}^2 - \hat{u}^2)}. \tag{5.5}$$

On the other hand, the Killing norm is proportional to Y (cf equation (36) in [19]) and thus nonextremal horizons are possible if $\tilde{u}$ and $\hat{u}$ are both non-zero and field-dependent[11] (e.g. $u = a + b\phi, \tilde{u} + \hat{u} = a + b\phi, \tilde{u} - \hat{u} = c$; $a, b, c \in \mathbb{R}$). However, following the arguments of [2] all BPS BHs should be extremal in generalized gPSM gravity theories as well[12].

This apparent contradiction is resolved by investigating singularity obstructions on the Poisson tensor (cf section 3 in [9] and [7]). Solving the Jacobi identity (2.3) with $u, \tilde{u}, \hat{u}$ and v as a given input, all remaining functions $P^{\alpha\beta}$, v_2 in (5.4) and $v_2^{\alpha\beta}$ are proportional to Δ^{-1}. Only for very special relations among the four free functions can the inverse powers of Δ be removed. It turns out that these relations imply extremality of eventual horizons appearing in BPS solutions. Consequently, 'BPS states' of generalized gPSM gravity theories with nonextremal horizons are *singular* solutions of the equations of motion[13].

6. Extension with conformal matter

We will prove in the following section that the conclusions of section 5.1 do not change when conformal matter is coupled to the dilaton SUGRA system. As these conclusions rely on the details of the symmetry transformations and the conserved quantities, in a first step the extension to MFS with matter fields is introduced in this section. To this end the close relation between MFS and the models obtained from superspace [8] is used. In superspace non-minimally coupled conformal matter is described by the Lagrangian

$$S_{(m)} = \frac{1}{4} \int d^2x \, d^2\theta \, E \, P(\Phi) D^\alpha M D_\alpha M. \tag{6.1}$$

Here $P(\Phi)$ is a function of the dilaton superfield Φ (cf (C.5)) and for the θ-expansion of the matter multiplet M we write[14]

$$M = f - i\theta\lambda + \tfrac{1}{2}\theta^2 H. \tag{6.2}$$

Integrating out superspace one arrives at (cf appendix C)

$$\begin{aligned}
S_{(m)} = \int d^2x \, e \bigg[& P(\phi) \left(\frac{1}{2}(\partial^m f \partial_m f + i\lambda\gamma^m \partial_m \lambda + H^2) \right. \\
& \left. + i(\psi_n \gamma^m \gamma^n \lambda)\partial_m f + \frac{1}{4}(\psi^n \gamma_m \gamma_n \psi^m)\lambda^2 \right) \\
& + \frac{1}{4} P'(\phi)(i(\lambda\gamma_*\chi)H - (\chi\gamma_*\gamma^m\lambda)\partial_m f - F\lambda^2) - \frac{1}{32}P''(\phi)\chi^2\lambda^2 \bigg].
\end{aligned} \tag{6.3}$$

[11] These states in general are not ground states in the sense of section 3.

[12] The constraints from gPSM symmetries are first class and free of ordering problems [7, 20, 21]. Therefore, on the constraint surface the unbroken fermionic symmetry still commutes with the Hamiltonian, which is the central ingredient in the argument by Gibbons.

[13] Similar states with singularities in the gravitino sector at the horizon had been found in 4D supergravity as well, cf the discussion in [2].

[14] Whenever the distinction between superfield components and MFS fields is important, underlined symbols are used for the former ones. However, for simplicity this is omitted in most formulae of this section, as the matter action is invariant under the redefinition (C.18), while all other identifications are trivial.

The action (6.3) depends on the auxiliary fields H from the matter multiplet and F from the dilaton multiplet. H can be integrated out without detailed knowledge of the geometric part of the action. To integrate out F, however, $u(\Phi)$ and $Z(\Phi)$ in (C.1) must be specified[15].

6.1. Matter extension at $\bar{Z} = 0$

A particularly simple situation is realized by choosing $\bar{Z} = 0$ (following the notation of [8], barred variables are used for this special case throughout). Then the action (C.1) is bilinear in $\bar{A}$ and $\bar{F}$ and the elimination condition (C.12) is modified according to $\bar{A} = -\bar{u}'/2 + \bar{P}'\bar{\lambda}^2/4$, $\bar{F} = -\bar{u}/2$. The part of the action independent of the matter field retains the form (C.13) with $\bar{Z} = 0$, while (6.3) after elimination of all auxiliary fields becomes

$$\bar{S}_{(m)} = \int d^2 x\, \bar{e}\left[\bar{P}\left(\frac{1}{2}(\partial^m \bar{f}\partial_m \bar{f} + i\bar{\lambda}\gamma^m \partial_m \bar{\lambda}) + i(\bar{\psi}_n\gamma^m \gamma^n \bar{\lambda})\partial_m \bar{f} + \frac{1}{4}(\bar{\psi}^n \gamma_m \gamma_n \bar{\psi}^m)\bar{\lambda}^2 \right) \right.$$
$$\left. + \frac{\bar{u}}{8}\bar{P}'\bar{\lambda}^2 - \frac{1}{4}\bar{P}'(\bar{\chi}\gamma_*\gamma^m\bar{\lambda})\partial_m \bar{f} - \frac{1}{32}\left(\bar{P}'' - \frac{1}{2}\frac{[\bar{P}']^2}{\bar{P}} \right)\bar{\chi}^2\bar{\lambda}^2 \right]. \tag{6.4}$$

The symmetry transformations of the matter fields $\bar{f}$ and $\bar{\lambda}_\alpha$ after elimination of $\bar{H}$ read (it should be noted [8] that the symmetry parameters ε and $\underline{\varepsilon}$ are different in general)

$$\bar{\delta}\bar{f} = i(\underline{\bar{\varepsilon}}\bar{\lambda}) \qquad \bar{\delta}\bar{\lambda}_\alpha = (\partial_m \bar{f} + i(\bar{\psi}_m\bar{\lambda}))(\gamma^m\underline{\varepsilon})_\alpha - \frac{1}{4}\frac{P'}{P}(\bar{\lambda}\gamma_*\bar{\chi})\underline{\bar{\varepsilon}}_\alpha \tag{6.5}$$

while the zweibein, the dilaton and the dilatino still transform according to (C.14), (C.16) and (C.17), resp. The transformation rule for the gravitino changes, as it depends on the auxiliary field $\bar{A}$:

$$\delta\underline{\bar{\psi}}_m{}^\alpha = -(\tilde{\bar{D}}\underline{\bar{\varepsilon}})^\alpha + \frac{i}{4}\left(\bar{u}' - \frac{1}{2}\bar{P}'\bar{\lambda}^2 \right)(\underline{\bar{\varepsilon}}\gamma_m)^\alpha. \tag{6.6}$$

When working with the MFS formulation of the geometric part, it is advantageous to formulate the matter action (6.4) in terms of differential forms as well:

$$S_{(m)} = \int_M \left[\bar{P}\left(\frac{1}{2}d\bar{f} \wedge *d\bar{f} + \frac{i}{2}\bar{\lambda}\gamma_a\bar{e}^a \wedge *d\bar{\lambda} + i*(\bar{e}_a \wedge *d\bar{f})\bar{e}_b \wedge *\bar{\psi}\gamma^a\gamma^b\bar{\lambda} \right. \right.$$
$$\left. + \frac{1}{4}*(\bar{e}_b \wedge *\bar{\psi})\gamma^a\gamma^b\bar{e}_a \wedge *\bar{\psi}\bar{\lambda}^2 \right) + \frac{\bar{u}}{8}\bar{P}'\bar{\lambda}^2\bar{\epsilon}$$
$$\left. - \frac{1}{4}\bar{P}'(\bar{\chi}\gamma_*\gamma^a\bar{\lambda})\bar{e}_a \wedge *d\bar{f} - \frac{1}{32}\left(\bar{P}'' - \frac{1}{2}\frac{[\bar{P}']^2}{\bar{P}} \right)\bar{\chi}^2\bar{\lambda}^2\bar{\epsilon} \right]. \tag{6.7}$$

For the special case $\bar{Z} = 0$ discussed so far, the identification (C.18) between the variables of MFS and of the superfield formulation (C.1) in [5] becomes trivial. Thus, replacing $\underline{\bar{\varepsilon}}$ in (6.5) and (6.6) by $\bar{\varepsilon}$, the action (2.18) with $\bar{Z} = 0$ together with (6.4) is invariant under (2.19)–(2.21), (6.5) and (6.6). The special case of minimal coupling ($P(\bar{\phi}) = 1$) of this matter extension of a gPSM-based dilaton SUGRA model has already been obtained in [11] using Noether techniques. But the above derivation using the equivalence of this theory to a superspace formulation has definite advantages as it straightforwardly generalizes to more complicated matter actions.

As a first step we should derive from the result obtained so far the matter extension of MFS$_0$ (MFS$_0$ indicates MFS for $\bar{Z} = 0$). This step is necessary as only the first-order formalism

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[15] As F appears in a term $\propto P'$ the restriction to $\bar{Z} = 0$ in the following is not necessary when considering minimally coupled matter.

in terms of a gPSM allows the straightforward treatment of the model at the classical as well as at the quantum level. Also in the present context the MFS formulation is much superior. The MFS action is different from (2.18), which was obtained after elimination of X^a and (the torsion-dependent part of) ω. Now these auxiliary variables must be re-introduced *together* with the matter coupling. Second, we would like to extend the matter coupling to the most general MFS (2.9). So far, we arrived at a matter extension for the special case $\bar{Z} = 0$ only. The general matter coupling ($Z \neq 0$) will be obtained by the use of a certain dilaton-dependent conformal transformation (cf [8, 9]). It is argued in the end that the same result could also have been derived in a different way.

The discussion of a consistent matter extension of MFS_0 considerably simplifies by observing that the action (6.4) or (6.7) does not change when the independent variables $\bar{X}^a$ and $\bar{\omega}$ are re-introduced. This is trivial for $\bar{\omega}$, as (6.4) does not contain the dependent spin connection $\tilde{\bar{\omega}}$. The independence of $\bar{X}^a$ is obvious as well [8, 9]: the elimination condition of $\bar{X}^a$ depends on derivatives acting on the dilaton field and the matter action does not contain such terms. Thus the matter extension of MFS_0 must be of the form[16]

$$\bar{\mathcal{S}}_{\text{tot}}(\bar{X}, \bar{A}, \bar{f}, \bar{\lambda}) = \mathcal{S}_{\text{MFS}_0}(\bar{X}, \bar{A}) + \bar{\mathcal{S}}_{(m)}(\bar{e}_a, \bar{\psi}_\alpha, \bar{\phi}, \bar{f}, \bar{\lambda}). \tag{6.8}$$

Not completely trivial is the derivation of the correct supersymmetry transformations. However, it is important to realize that (6.8) already is invariant up to equations of motion[17] of $\bar{X}^a$ and $\bar{\omega}$: as these e.o.m.-s are linear in $\bar{X}^a$ and $\bar{\omega}$, the elimination of these fields 'commutes' with the symmetry transformation. Therefore there exists a simple and systematic way to modify the MFS_0 symmetry laws (2.12)–(2.17) such that (6.8) together with (6.5) is again invariant. In an abstract notation the behaviour of (6.8) under (2.12)–(2.17) and (6.5) may be written as

$$\bar{\delta}\bar{\mathcal{S}}_{\text{tot}}(\bar{X}, \bar{A}, \bar{f}, \bar{\lambda}) = N_X^a(\bar{X}, \bar{A}, \bar{f}, \bar{\lambda}; \bar{\varepsilon}) \cdot (\bar{X}^a\text{-e.o.m.}) + (\bar{\omega}\text{-e.o.m.}) \wedge N_\omega(\bar{X}, \bar{A}, \bar{f}, \bar{\lambda}; \bar{\varepsilon}). \tag{6.9}$$

Of course, the two field-dependent quantities N_X^a and N_ω multiplying the e.o.m.-s must vanish in the absence of matter fields:

$$N_X^a(\bar{X}, \bar{A}, \bar{f} = 0, \bar{\lambda} = 0; \bar{\varepsilon}) = 0 \qquad N_\omega(\bar{X}, \bar{A}, \bar{f} = 0, \bar{\lambda} = 0; \bar{\varepsilon}) = 0. \tag{6.10}$$

They are used to modify the symmetry transformations of $\bar{X}^a$ and $\bar{\omega}$ by

$$\bar{\delta}\bar{X}^a = \bar{\delta}_{\text{MFS}_0}\bar{X}^a - N_X^a \qquad \bar{\delta}\bar{\omega} = \bar{\delta}_{\text{MFS}_0}\bar{\omega} - N_\omega \tag{6.11}$$

where the transformations (2.12)–(2.17) with $\bar{Z} = 0$ have been renamed $\bar{\delta}_{\text{MFS}_0}$.

The explicit calculation of N_X^a and N_ω is straightforward. As $\bar{\mathcal{S}}_{(m)}$ depends on the MFS_0 fields $\bar{\phi}$, $\bar{e}_a$ and $\bar{\psi}_\alpha$ only, the variations (2.12), (2.16) and (2.17) within (6.4) lead to potential non-invariance. But (2.12) and (2.16) are equivalent to the supersymmetry transformations of these fields within the superfield formulation (cf (C.6), (C.9)). There remains the supersymmetry transformation of the gravitino. Again, most terms are equivalent to the superspace formulation (remember that $\bar{Z} = 0$!), except for the covariant derivative $(D\varepsilon)_\alpha$. Here the dependent spin connection $\tilde{\omega}$ has been replaced by the independent one. As the independent part of the spin connection is eliminated by means of the $\bar{X}^a$-e.o.m. (B.7) this leads to contributions to N_X^a. A further source of non-invariance is the modification of the gravitino transformation in (6.6). This yields another contribution to N_X^a from the covariant derivative acting on $\bar{\psi}$, but also to N_ω from the term $\propto \bar{X}^a \bar{\psi} \gamma_a \bar{\psi}$. The latter contributions are

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[16] $X^I = (\phi, X^a, \chi^\alpha)$, $A_I = (\omega, e_a, \psi_\alpha)$.
[17] We denote the equations of motion according to the field which has been varied. Thus the X^a-e.o.m. refers to (B.7), while the ω-e.o.m. refers to (B.3).

proportional to $P'(\bar{\phi})$ and vanish in the case of minimal coupling. Putting all terms together one finds

$$\bar{\delta}\bar{X}^a = \bar{\delta}_{\mathrm{MFS}_0}\bar{X}^a + \frac{\mathrm{i}}{2}P(\bar{\varepsilon}\gamma^m\gamma^a\gamma_*\bar{\lambda})\partial_m\bar{f} + \frac{1}{4}P(\bar{\varepsilon}\gamma^m\gamma^a\gamma_*\bar{\psi}_m)\bar{\lambda}^2 - \frac{\mathrm{i}}{16}P'(\bar{\chi}\gamma^a\bar{\varepsilon})\bar{\lambda}^2 \qquad (6.12)$$

$$\bar{\delta}\bar{\omega} = \bar{\delta}_{\mathrm{MFS}_0}\bar{\omega} - \frac{1}{4}P'(\bar{\varepsilon}\gamma_*\bar{\psi} - \bar{\varepsilon}^\alpha * \bar{\psi}_\alpha)\bar{\lambda}^2. \qquad (6.13)$$

With these new transformation laws the action (6.8) is finally fully invariant under supersymmetry, while local Lorentz invariance and diffeomorphism invariance are manifest.

6.2. Matter extension at $Z \neq 0$

To extend the matter couplings to the general MFS ($Z \neq 0$ in (2.9)) we use the conformal transformations (2.10) and (2.11) of section 2. The matter action is invariant under those transformations of the fields, when the new matter fields are defined as

$$f = \bar{f} \qquad \lambda = \mathrm{e}^{-\frac{1}{4}Q(\phi)}\bar{\lambda}. \qquad (6.14)$$

After the combined transformations (2.10), (2.11) and (6.14) an action with general MFS as geometrical part coupled to conformal matter is obtained. $\bar{\mathcal{S}}_{(m)}$ in (6.4) by construction is invariant under the conformal transformation and therefore that equation, after dropping all bars, is the correct matter extension of MFS. Of course, the new action

$$\mathcal{S}_{\mathrm{tot}}(X, A, f, \lambda) = \mathcal{S}_{\mathrm{MFS}}(X, A) + \mathcal{S}_{(m)}(e_a, \psi_\alpha, \phi, f, \lambda) \qquad (6.15)$$

is invariant under the *old* $\bar{\varepsilon}$-transformations that act on the barred variables

$$\bar{\delta}\mathcal{S}_{\mathrm{tot}}(X(\bar{X}), A(\bar{A}, \bar{X}), f(\bar{f}), \lambda(\bar{\lambda}, \bar{X})) = 0 \qquad (6.16)$$

but we have to be careful with the new transformations δ (depending on ε), as the transformation parameters themselves change under a conformal transformation as well. The importance of this behaviour for the understanding of gPSM-based SUGRA has been pointed out in [8]. Conformal transformations represent a special case of target space diffeomorphisms in the (g)PSM formulation (cf section 4.1 of [8]). Under such transformations the variables and symmetry parameters change as

$$\bar{\delta}\bar{X}^I = \delta\bar{X}^I(X) \qquad (6.17)$$

$$\bar{\delta}\bar{A}_I = \delta\bar{A}_I(A, X) + \text{e.o.m.-s} \qquad (6.18)$$

$$\bar{\varepsilon}_I = \frac{\partial X^J}{\partial \bar{X}^I}\varepsilon_J \qquad (6.19)$$

where the indices are those used in the gPSM formulation (2.1). Equation (6.19) together with (2.10) and (2.11) for a pure supersymmetry transformation yield (cf section 5.2 in [8], especially equation (5.8)):

$$\bar{\varepsilon} = (\bar{\varepsilon}_\phi, \bar{\varepsilon}_a, \bar{\varepsilon}_\alpha) = (0, 0, \bar{\varepsilon}_\alpha) \xrightarrow{\text{conformal transformation}} \varepsilon = \left(\frac{Z}{4}(\chi\varepsilon), 0, \varepsilon_\alpha\right). \qquad (6.20)$$

Thus for the general MFS the symmetries (6.5) are modified by a local Lorentz transformation with field-dependent parameter $\varepsilon_\phi = \frac{1}{4}Z\chi\varepsilon$. The symmetry law of f remains unchanged under both the conformal transformation (6.14) and the additional local Lorentz transformation (6.20), as this field is invariant under these symmetries. However, for λ the local Lorentz transformation and the supersymmetry transformation of the conformal factor in (6.14) add up to the new contributions displayed in equation (6.27) below.

Still the action (6.15) is not invariant under (6.26), (6.27) and (2.12)–(2.17): first, the modified laws of $\bar\psi$ (6.6), $\bar{X}^a$ (6.12) and $\bar\omega$ (6.13) should be rewritten in terms of the MFS variables. But as none of these extensions generates derivatives onto the conformal factors, this boils down to rewriting these transformation rules in terms of variables without bars. Second, the e.o.m.-s appearing on the rhs of (6.18) may necessitate further modifications of the MFS symmetries. As the conformal transformations (2.10) and (2.11) depend on the dilaton field only, under supersymmetry transformations discussed so far the action (6.15) is invariant up to e.o.m.-s of ω. These new non-invariant terms originate from the variation of the gravitino (cf equation (4.8) of [8] and comments below this equation). Indeed a straightforward calculation shows that

$$\bar\delta \psi_\alpha = -\mathrm{d}\bar\epsilon_\alpha + \tfrac{1}{4} Z \,\mathrm{d}\phi \,\bar\epsilon_\alpha + \cdots \qquad \delta \psi_\alpha = -\mathrm{d}\epsilon_\alpha + \cdots \tag{6.21}$$

where the dots indicate terms which do not contain derivatives. The equation of motion of ω involved here drops out in the geometric part, but obviously not in the matter extension. However, from the above equation together with the formulation of $\mathcal{S}(m)$ in (6.7), supersymmetry can be restored analogously to the arguments leading to (6.12). It can be read off from (6.21) that the new (matter-field dependent) piece to the transformation of ω is obtained by replacing ψ_α in (6.7) by $\tfrac{1}{4} Z\epsilon_\alpha$:

$$\delta \omega = (6.13) - \tfrac{1}{4} Z P \left(\mathrm{i}(\varepsilon \gamma^a \gamma^b \lambda) e_a^m \partial_m f (*e_b) + \tfrac{1}{2}(\varepsilon \gamma^a \gamma^b \psi_m) e_a^m (*e_b)\lambda^2 \right). \tag{6.22}$$

It is useful to summarize what we have obtained in section 6: the gPSM-based MFS models of equation (2.9) can be extended by the coupling of matter fields. The complete action (6.15) is given by the sum of (2.9) and (6.4). The supersymmetry transformations (2.12), (2.14), (2.16) for ϕ, χ^α and e_a are not changed by the matter coupling. Equation (2.17) for the gravitino receives new contributions from the elimination of the auxiliary fields in superspace, while δX^a and $\delta \omega$ are changed by re-introducing the auxiliary fields of the gPSM formulation. Thus, the complete list of supersymmetry transformations is given by (2.12)–(2.17) plus new contributions from the matter fields,

$$\delta_{(m)} X^a = \frac{\mathrm{i}}{2} P(\varepsilon \gamma^m \gamma^a \gamma_* \lambda)\partial_m f + \frac{1}{4} P(\varepsilon \gamma^m \gamma^a \gamma_* \psi_m)\lambda^2 - \frac{\mathrm{i}}{16} P'(\chi \gamma^a \epsilon)\lambda^2 \tag{6.23}$$

$$\delta_{(m)}\omega = -\frac{1}{4} P'(\epsilon \gamma_* \psi - \epsilon^\alpha * \psi_\alpha)\lambda^2$$
$$\qquad\qquad - \frac{1}{4} Z P \left(\mathrm{i}(\varepsilon \gamma^a \gamma^b \lambda) e_a^m \partial_m f (*e_b) + \frac{1}{2}(\varepsilon \gamma^a \gamma^b \psi_m) e_a^m (*e_b)\lambda^2 \right) \tag{6.24}$$

$$\delta_{(m)}\psi_\alpha = \mathrm{i} \frac{P'}{8} \lambda^2 (\gamma^b \varepsilon)_\alpha e_b \tag{6.25}$$

together with the transformations of the matter fields

$$\delta f = \mathrm{i}(\varepsilon \lambda) \tag{6.26}$$

$$\delta \lambda_\alpha = (\partial_m f + \mathrm{i}(\psi_m \lambda))(\gamma^m \varepsilon)_\alpha - \frac{1}{8} Z((\chi \gamma_* \varepsilon)\lambda_\alpha + (\chi \varepsilon)(\gamma_* \lambda)_\alpha) - \frac{P'}{4P}(\lambda \gamma_* \chi)\varepsilon_\alpha. \tag{6.27}$$

Of course, we could have used the relation to the general Park–Strominger model SFDS of equation (C.13) to the MFS (cf (C.18) and also [8]) instead of the conformal transformation of MFS_0 to derive the matter coupling at $Z \neq 0$. Thus we may eliminate X^a and ω in (6.15) and by this procedure arrive at the matter action (6.4) coupled to MFDS. Using

the techniques developed in [8] this equivalence follows almost trivially. Considering the symmetry transformations we note that we find again (cf (C.19))

$$\Delta\lambda_\alpha = -\tfrac{1}{4}Z(\chi\varepsilon)(\gamma_*\lambda)_\alpha \tag{6.28}$$

in agreement with the result derived in [8].

One might wonder whether, on a different route, it is possible to derive a different matter extension of gPSM-based SUGRA, where modifications of the transformation laws of X^I and A_I do not occur. The answer is negative, as long as this extension shall preserve both local Lorentz invariance and supersymmetry. Indeed, the commutator of two local supersymmetry transformations is a local Lorentz transformation δ_ϕ plus a 'local translation' δ_a. Invariance under strict gPSM symmetry transformations would imply that the matter action is invariant under δ_a, which, except for rigid supersymmetry, cannot be fulfilled.

7. Supersymmetric ground states with matter

The matter extension of MFS derived in the previous section allows the discussion of supersymmetric ground states including matter fields. The fully supersymmetric states are trivial: the geometric variables obey the same constraints as derived already in section 3, the matter fields must obey $f = \text{const}$ and $\lambda = 0$.

More involved are the states with one supersymmetry: equation (6.26) leads to

$$\lambda^+\varepsilon_+ = -\lambda^-\varepsilon_- \tag{7.1}$$

which, in analogy to (4.1), implies $\lambda^2\varepsilon \equiv 0$. Furthermore, as (2.12), (2.14) and (2.16) did not receive new matter-field-dependent contributions, the relations (4.1), (4.2) and (4.3) still hold. Of course, this still implies (4.4), but the geometric part of the Casimir function is no longer conserved (see discussion below).

As a consequence of (7.1) and (4.1)–(4.3) the vanishing of $\delta\lambda_\alpha$ in (6.27) reduces to

$$\delta\lambda_\alpha = (\gamma^m\varepsilon)_\alpha\partial_m f = 0. \tag{7.2}$$

It is straightforward to check that all matter-field-dependent modifications in (6.23)–(6.25) vanish due to (7.1) and (7.2). Thus the matter couplings do not change the classification of the solutions as given in section 4 as well as the results of section 5.

In order to understand the condition (7.2) on the matter-field configurations it is advantageous to reformulate it as

$$f^{++}\varepsilon_+ = 0 \qquad f^{--}\varepsilon_- = 0 \tag{7.3}$$

where $f^{\pm\pm} = *(e^{\pm\pm}\wedge\mathrm{d}f)$ are (anti-)selfdual field configurations of the scalar field. Thus the chiral solution of (4.1) and (7.1) with $\chi^- = \lambda^- = \varepsilon_+ = 0$ admits selfdual scalar fields while the anti-chiral one allows the anti-selfdual f. The third solution with $\varepsilon_+ \neq 0$ and $\varepsilon_- \neq 0$ is compatible with static f, only.

Even in the presence of matter a conserved quantity can be constructed. Its physical relevance is displayed in the close relationship to energy definitions well-known from general relativity, such as ADM-, Bondi- and quasi-local mass (for details we refer to section 5 of [16] and references therein).

The conservation law $\mathrm{d}C = 0$ in the presence of matter fields is modified by analogy to the pure bosonic case (cf [22]) according to

$$e^{-Q}\mathrm{d}C + X^{--}W^{++} + X^{++}W^{--} + \frac{1}{8}\left(u' + \frac{1}{2}uZ\right)(\chi^- W_- - \chi^+ W_+) = 0 \tag{7.4}$$

$$W^{\pm\pm} = \frac{\delta}{\delta e^{\mp\mp}}\mathcal{S}_{(m)} \qquad W_\pm = \frac{\delta}{\delta\psi^\mp}\mathcal{S}_{(m)}. \tag{7.5}$$

In the presence of matter $(-W^{\pm\pm})$ appear on the rhs of the e.o.m.-s (B.4), $(-W_\pm)$ on the rhs of (B.5). Equation (7.4) results from a suitable linear combination of the e.o.m.-s (B.3)–(B.5), C now is only *part* of a total conserved quantity, which also contains a matter contribution $e^{-Q}\mathrm{d}C_{(m)}$ from the W terms.

A straightforward calculation from equation (6.4) yields

$$
\begin{aligned}
W^{\pm\pm} = \pm e^{\pm\pm}&\left[P\left(f^{++}f^{--} + \mathrm{i}*(e^{++}\psi\lambda)f^{--} + \mathrm{i}*(e^{--}\psi\lambda)f^{++}\right.\right.\\
&\left.+\frac{1}{2}*(\psi e^{--})^\alpha*(\psi e^{++})_\alpha(\lambda\lambda)\right) + \frac{u}{8}P'\lambda^2 + \frac{1}{32}\left(P'' - \frac{1}{2}\frac{P'^2}{P}\right)\chi^2\lambda^2\right]\\
&+\mathrm{d}f\left[P(f^{\pm\pm} + \mathrm{i}*(e^{\pm\pm}\psi\lambda)) - \frac{\mathrm{i}}{2\sqrt{2}}P'\chi^\pm\lambda^\pm\right]\\
&+P\left[\mathrm{i}(\psi\lambda)f^{\pm\pm} - \frac{1}{2}\psi*(\psi e^{\pm\pm})(\lambda\lambda) \mp \frac{1}{\sqrt{2}}\lambda^\pm\,\mathrm{d}\lambda^\pm\right]
\end{aligned}\tag{7.6}
$$

$$
\begin{aligned}
W_\mp = P&\left[-\mathrm{i}\lambda^\pm(e^{++}f^{--} + e^{--}f^{++} \pm \mathrm{d}f) - \frac{1}{2}\psi_\mp(\lambda\lambda)\right.\\
&\left.\pm\frac{1}{2}(e^{++}*(\psi_\mp e^{--}) + e^{--}*(\psi_\mp e^{++}))(\lambda\lambda)\right].
\end{aligned}\tag{7.7}
$$

Now also the question may be posed about the meaning of the restriction (4.4) within that generalized conservation law. The body of $\mathrm{d}C$ in (7.4) vanishes trivially due to that equation. The restriction to (anti-)selfdual or static f as derived from (7.3) ensures that the body of (7.4) vanishes without imposing further constraints on the fields. Thus the result of section 5 for the matterless case continues to hold if non-minimally coupled conformal matter is included.

8. Conclusions

In our present work we present the complete classification of all BPS BHs in 2D dilaton SUGRA coupled to conformal matter. The use of a first-order formulation as suggested from the graded Poisson–Sigma model approach for the geometric part of the action plays a crucial role in the calculations. As no matter extension thereof had been considered in the literature, its derivation is an important result on its own. For future reference we compile the MFS action non-minimally coupled to conformal matter (with coupling function $P(\phi)$) at this place:

$$
\begin{aligned}
\mathcal{S} = \int_{\mathcal{M}}&\left[\phi\,\mathrm{d}\omega + X^a De_a + \chi^\alpha D\psi_\alpha + \epsilon\left(V + YZ - \frac{1}{2}\chi^2\left(\frac{VZ+V'}{2u} + \frac{2V^2}{u^3}\right)\right)\right.\\
&+\frac{Z}{4}X^a(\chi\gamma_a\gamma^b e_b\gamma_*\psi) + \frac{\mathrm{i}V}{u}(\chi\gamma^a e_a\psi) + \mathrm{i}X^a(\psi\gamma_a\psi) - \frac{1}{2}\left(u + \frac{Z}{8}\chi^2\right)(\psi\gamma_*\psi)\\
&+P\left(\frac{1}{2}\mathrm{d}f \wedge *\mathrm{d}f + \frac{\mathrm{i}}{2}\lambda\gamma_a e^a \wedge *\mathrm{d}\lambda + \mathrm{i}*(e_a \wedge *\mathrm{d}f)e_b \wedge *\psi\gamma^a\gamma^b\lambda\right.
\end{aligned}
$$

$$+ \frac{1}{4} * (e_b \wedge *\psi)\gamma^a \gamma^b e_a \wedge *\psi \lambda^2 \Big) + \frac{u}{8} P' \lambda^2 \epsilon$$

$$- \frac{1}{4} P'(\chi \gamma_* \gamma^a \lambda) e_a \wedge *\mathrm{d}f - \frac{1}{32}\left(P'' - \frac{1}{2}\frac{[P']^2}{P}\right)\chi^2 \lambda^2 \epsilon \Bigg]. \tag{8.1}$$

This action is invariant under the supersymmetry transformations (2.12)–(2.17) supplemented by (6.23)–(6.27) and (6.26), (6.27).

Starting from this action it has been shown that all BPS-like states have vanishing body of the Casimir function and thus are ground states. Solutions with vanishing fermions allow a nontrivial bosonic geometry, but all Killing horizons were found to be extremal. On the other hand, the geometry of solutions with nonvanishing fermions must be Minkowski space and consequently there exist no supersymmetric BHs with dilatino or gravitino hair. The impossibility of supersymmetric dS ground states has been reproduced for our class of models and the absolute conservation law—the modification of the Casimir function in the presence of matter fields—has been calculated explicitly.

Note added in proof. While proof reading, an e-print appeared [25] which allows a nice application of some of the current paper's methods. That study is based upon 2D type 0A string theory and among other issues an upper bound on the number $q \leqslant 16\pi e < 12$ of electric and magnetic D0 branes is derived (equation (4.7) of [25]). The same bound immediately follows from reality of the prepotential

$$u(\phi) \propto \sqrt{1 - (q^2/(16\pi))(\ln\phi/\phi)}$$

or, equivalently, from semi-negativity of $w(\phi)$ in (B.12). Note that our dilaton ϕ is related to the dilaton Φ in [25] by $\phi = \exp(-2\Phi)$. In addition, as a simple consequence of the conservation of the Casimir function (B.9) we agree on the result for the ADM mass (equation (3.9) of [25]).

Appendix A. Notation and conventions

These conventions are identical to [9, 23], where additional explanations can be found.

Indices chosen from the Latin alphabet are commuting (lower case) or generic (upper case), Greek indices are anti-commuting. Holonomic coordinates are labelled by M, N, O etc, anholonomic ones by A, B, C etc, whereas I, J, K etc are general indices of the gPSM. The index ϕ is used to indicate the dilaton component of the gPSM fields:

$$X^\phi = \phi \qquad A_\phi = \omega. \tag{A.1}$$

The summation convention is always $NW \to SE$, e.g. for a fermion χ: $\chi^2 = \chi^\alpha \chi_\alpha$. Our conventions are arranged in such a way that almost every bosonic expression is transformed trivially to the graded case when using this summation convention and replacing commuting indices by general ones. This is possible together with exterior derivatives acting *from the right*, only. Thus the graded Leibniz rule is given by

$$\mathrm{d}(AB) = A\mathrm{d}B + (-1)^B (\mathrm{d}A)B. \tag{A.2}$$

In terms of anholonomic indices the metric and the symplectic 2×2 tensor are defined as

$$\eta_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \epsilon_{ab} = -\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \qquad (A.3)$$

The metric in terms of holonomic indices is obtained by $g_{mn} = e_n^b e_m^a \eta_{ab}$ and for the determinant the standard expression $e = \det e_m^a = \sqrt{-\det g_{mn}}$ is used. The volume form reads $\epsilon = \frac{1}{2}\epsilon^{ab} e_b \wedge e_a$; by definition $*\epsilon = 1$.

The γ-matrices are used in a chiral representation:

$$\gamma^0{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^1{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \gamma_{*\alpha}{}^\beta = (\gamma^1\gamma^0)_\alpha{}^\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \qquad (A.4)$$

Covariant derivatives of anholonomic indices with respect to the geometric variables $e_a = dx^m e_{am}$ and $\psi_\alpha = dx^m \psi_{\alpha m}$ include the two-dimensional spin-connection one form $\omega^{ab} = \omega\epsilon^{ab}$. When acting on lower indices the explicit expressions read ($\frac{1}{2}\gamma_*$ is the generator of Lorentz transformations in spinor space):

$$(De)_a = de_a + \omega\epsilon_a{}^b e_b \qquad (D\psi)_\alpha = d\psi_\alpha - \frac{1}{2}\omega\gamma_{*\alpha}{}^\beta\psi_\beta. \qquad (A.5)$$

Light-cone components are very convenient. As we work with spinors in a chiral representation we can use

$$\chi^\alpha = (\chi^+, \chi^-) \qquad \chi_\alpha = \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix}. \qquad (A.6)$$

For Majorana spinors upper and lower chiral components are related by $\chi^+ = \chi_-, \chi^- = -\chi_+, \chi^2 = \chi^\alpha\chi_\alpha = 2\chi_-\chi_+$. Vectors in light-cone coordinates are given by

$$v^{++} = \frac{i}{\sqrt{2}}(v^0 + v^1) \qquad v^{--} = \frac{-i}{\sqrt{2}}(v^0 - v^1). \qquad (A.7)$$

The additional factor i in (A.7) permits a direct identification of the light-cone components with the components of the spin-tensor $v^{\alpha\beta} = \frac{i}{\sqrt{2}}v^c\gamma_c{}^{\alpha\beta}$. This implies that $\eta_{++|--} = 1$ and $\epsilon_{--|++} = -\epsilon_{++|--} = 1$.

Appendix B. E.o.m.-s and conserved quantity

The equations of motion for a generic gPSM are

$$dX^I + P^{IJ}A_J = 0 \qquad (B.1)$$

$$dA_I + \frac{1}{2}(\partial_I P^{JK})A_K A_J = 0. \qquad (B.2)$$

Consequently, those for the MFS action (2.9) become

$$d\phi - X^b\epsilon_b{}^a e_a + \frac{1}{2}(\chi\gamma_*\psi) = 0 \qquad (B.3)$$

$$DX^a + \epsilon^{ab}e_b\left(V + YZ - \frac{1}{2}\chi^2\left(\frac{VZ + V'}{2u} + \frac{2V^2}{u^3}\right)\right) - \frac{Z}{4}X^b(\chi\gamma_b\gamma^a\gamma_*\psi) - \frac{iV}{u}(\chi\gamma^a\psi)$$
$$= 0 \qquad (B.4)$$

$$D\chi^\alpha + \frac{Z}{4}X^a(\chi\gamma_a\gamma^b\gamma_*)^\alpha e_b + \frac{iV}{u}(\chi\gamma^a)^\alpha e_a + 2iX^a(\psi\gamma_a)^\alpha - \left(u + \frac{Z}{8}\chi^2\right)(\psi\gamma_*)^\alpha = 0 \quad (B.5)$$

$$d\omega + \epsilon\left(V' + YZ' - \frac{1}{2}\chi^2\left(\left(\frac{VZ + V'}{2u}\right)' + \left(\frac{2V^2}{u^3}\right)'\right)\right) + \frac{Z'}{4}X^b(\chi\gamma_b\gamma^a e_a\gamma_*\psi)$$

$$+ i\left(\frac{V}{u}\right)'(\chi\gamma^a e_a\psi) - \frac{1}{2}\left(u' + \frac{Z'}{8}\chi^2\right)(\psi\gamma_*\psi) = 0 \tag{B.6}$$

$$De_a + \eta_{ab}\epsilon X^b Z + \frac{Z}{4}(\chi\gamma_a\gamma^b e_b\gamma_*\psi) + i(\psi\gamma_a\psi) = 0 \tag{B.7}$$

$$D\psi_\alpha - \epsilon\chi_\alpha\left(\frac{VZ + V'}{2u} + \frac{2V^2}{u^3}\right) + \frac{Z}{4}X^a(\gamma_a\gamma^b e_b\gamma_*\psi)_\alpha + \frac{iV}{u}(\gamma^a e_a\psi)_\alpha - \frac{Z}{8}\chi_\alpha(\psi\gamma_*\psi)$$
$$= 0. \tag{B.8}$$

We re-emphasize that V, Z and the prepotential u are related by (2.8).

The full analytic solution of MFS has been given in section 6 of [8]. Each solution is characterized by a certain value of the Casimir function, a quantity conserved in space and time. It consists of a bosonic part (body) and a fermionic one (soul):

$$C = C_B + C_S \tag{B.9}$$

$$C_B = e^{Q(\phi)}Y + w(\phi) = e^{Q(\phi)}\left(Y - \frac{1}{8}u^2(\phi)\right) \tag{B.10}$$

$$C_S = \frac{1}{16}e^Q\chi^2\left(u' + \frac{1}{2}uZ\right). \tag{B.11}$$

In this equation the (logarithm of the) integrating factor and the conformally invariant combination of the bosonic potentials

$$Q(\phi) := \int^\phi Z(\phi')\,d\phi' \qquad w(\phi) := \int^\phi e^{Q(\phi')}V(\phi')\,d\phi' = -\frac{1}{8}e^{Q(\phi)}u^2(\phi) \leqslant 0 \tag{B.12}$$

have been introduced. In [8] the solutions for $C \neq 0$ (equations (6.9)–(6.13)) and $C = 0$ (equations (6.17)–(6.21)) have been given which are not reproduced here.

Appendix C. Dilaton SUGRA in superspace

The action for a general dilaton SUGRA in superspace [5] may be written as[18]

$$\mathcal{S}_{\text{SFDS}} = \int d^2x\,d^2\theta\,E\left(\Phi S - \frac{1}{4}Z(\Phi)D^\alpha\Phi D_\alpha\Phi + \frac{1}{2}u(\Phi)\right) \tag{C.1}$$

with[19] [24]

$$E = e(1 + i\theta\gamma^a\underline{\psi}_a + \frac{1}{2}\theta^2(\underline{A} + \epsilon^{ab}\underline{\psi}_b\gamma_*\underline{\psi}_a) \tag{C.2}$$

$$S = \underline{A} + 2\theta\gamma_*\underline{\tilde{\sigma}} - i\underline{A}\theta\gamma^a\underline{\psi}_a + \frac{1}{2}\theta^2(\epsilon^{mn}\partial_n\underline{\tilde{\omega}}_m - \underline{A}(\underline{A} + \epsilon^{ab}\underline{\psi}_b\gamma_*\underline{\psi}_a) - 2i\underline{\psi}^a\gamma_a\gamma_*\underline{\tilde{\sigma}}) \tag{C.3}$$

$$D_\alpha = \partial_\alpha + i(\gamma^a\theta)_\alpha\partial_a. \tag{C.4}$$

Quantities with a tilde are defined in analogy to footnote 3. Φ is the dilaton superfield with component expansion

$$\Phi = \underline{\phi} + \frac{1}{2}\theta\gamma_*\underline{\chi} + \frac{1}{2}\theta^2\underline{F}. \tag{C.5}$$

[18] In [5] the first term in the brackets was chosen as $EJ(\Phi)S$. If a global field redefinition $J(\Phi) \to \Phi$ is not possible, these models are not equivalent globally to MFS [8].

[19] Except for the zweibein, components of superfields are denoted by underlined variables to distinguish them from the fields in the gPSM approach.

The supersymmetry transformations of the component fields of the superdeterminant are given by

$$\delta e_m{}^a = -2\mathrm{i}(\underline{\varepsilon}\gamma^a\underline{\psi}_m) \qquad \delta e^m{}_a = 2\mathrm{i}(\underline{\varepsilon}\gamma^m\underline{\psi}_a) \tag{C.6}$$

$$\delta\underline{\psi}_m{}^\alpha = -\left((\tilde{\underline{D}}\underline{\varepsilon})^\alpha + \frac{\mathrm{i}}{2}\underline{A}(\underline{\varepsilon}\gamma_m)^\alpha\right) \tag{C.7}$$

$$\delta\underline{A} = -2\left((\underline{\varepsilon}\gamma_*\tilde{\underline{\sigma}}) - \frac{\mathrm{i}}{2}\underline{A}e^m{}_a(\underline{\varepsilon}\gamma^a\underline{\psi}_m)\right) \tag{C.8}$$

while those of the dilaton superfield read

$$\delta\underline{\phi} = -\tfrac{1}{2}\underline{\varepsilon}\gamma_*\underline{\chi} \tag{C.9}$$

$$\delta\underline{\chi}_\alpha = -2(\gamma_*\underline{\varepsilon})_\alpha\underline{F} + \mathrm{i}(\gamma_*\gamma^b\underline{\varepsilon})_\alpha(\underline{\psi}_b\gamma_*\underline{\chi}) - 2\mathrm{i}(\gamma_*\gamma^m\underline{\varepsilon})_\alpha\partial_m\underline{\phi} \tag{C.10}$$

$$\delta\underline{F} = \mathrm{i}(\underline{\varepsilon}\gamma^a\underline{\psi}_a)\underline{F} - \frac{\mathrm{i}}{2}(\underline{\varepsilon}\gamma^m\gamma_*(\tilde{\underline{D}}_m\underline{\chi})) + (\underline{\varepsilon}\underline{\lambda}^m)((\underline{\psi}_m\gamma_*\underline{\chi}) - 2\partial_m\underline{\phi}). \tag{C.11}$$

Integrating out superspace and eliminating the auxiliary fields $\underline{A}$ and $\underline{F}$ using their equations of motion

$$\underline{F} = -\frac{u}{2} \qquad \underline{A} = -\frac{1}{2}(u' + uZ) + \frac{1}{8}Z'\underline{\chi}^2. \tag{C.12}$$

one arrives at the action

$$\begin{aligned}
\mathcal{S}_{\mathrm{SFDS}} = \int \mathrm{d}^2x\, e\Bigg(&\frac{1}{2}\tilde{\underline{R}}\underline{\phi} + (\underline{\chi}\tilde{\underline{\sigma}}) - \frac{1}{2}Z\Bigg(\partial^m\underline{\phi}\partial_m\underline{\phi} - \frac{\mathrm{i}}{4}\underline{\chi}\gamma^m\partial_m\underline{\chi} \\
&- (\underline{\psi}_n\gamma^m\gamma^n\gamma_*\underline{\chi})\partial_m\underline{\phi}\Bigg) - \frac{1}{8}((u^2)' + u^2 Z) + \frac{u}{2}\epsilon^{mn}(\underline{\psi}_n\gamma_*\underline{\psi}_m) \\
&+ \frac{\mathrm{i}}{2}u'(\underline{\zeta}\gamma_*\underline{\chi}) + \frac{1}{8}\left(u'' + \frac{1}{4}uZ' + \frac{1}{2}Z(\underline{\psi}_n\gamma^m\gamma^n\underline{\psi}_m)\right)(\underline{\chi}\underline{\chi})\Bigg)
\end{aligned} \tag{C.13}$$

while the symmetry transformations of the remaining fields take the form

$$\delta e_m{}^a = -2\mathrm{i}(\underline{\varepsilon}\gamma^a\underline{\psi}_m) \qquad \delta e^m{}_a = 2\mathrm{i}(\underline{\varepsilon}\gamma^m\underline{\psi}_a) \tag{C.14}$$

$$\delta\underline{\psi}_m{}^\alpha = -(\tilde{D}\underline{\varepsilon})^\alpha + \frac{\mathrm{i}}{4}\left(u' + uZ - \frac{1}{8}Z'(\underline{\chi}\underline{\chi})\right)(\underline{\varepsilon}\gamma_m)^\alpha \tag{C.15}$$

$$\delta\underline{\phi} = -\frac{1}{2}\underline{\varepsilon}\gamma_*\underline{\chi} \tag{C.16}$$

$$\delta\underline{\chi}_\alpha = u(\gamma_*\underline{\varepsilon})_\alpha + \mathrm{i}(\gamma_*\gamma^b\underline{\varepsilon})_\alpha(\underline{\psi}_b\gamma_*\underline{\chi}) - 2\mathrm{i}(\gamma_*\gamma^m\underline{\varepsilon})_\alpha\partial_m\underline{\phi}. \tag{C.17}$$

In [8] it has been shown that this action is equivalent to the action (2.18) of MFDS if the identifications

$$\underline{\psi}_m{}^\alpha = \psi_m^\alpha + \frac{\mathrm{i}}{8}Z(\phi)e_m^a\epsilon_{ab}(\chi\gamma^b)^\alpha \qquad \underline{\phi} = \phi \qquad \underline{\chi} = \chi \tag{C.18}$$

are made. The supersymmetry transformations are equivalent up to a local Lorentz transformation with field-dependent parameter:

$$\varepsilon = \underline{\varepsilon} \qquad \Delta = \delta_{\mathrm{MFDS}} - \delta_{\mathrm{SFDS}} = \frac{Z}{2}\chi\varepsilon\delta_\phi. \tag{C.19}$$

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